Computer Generated Hologram Optimization for Lens Aberration

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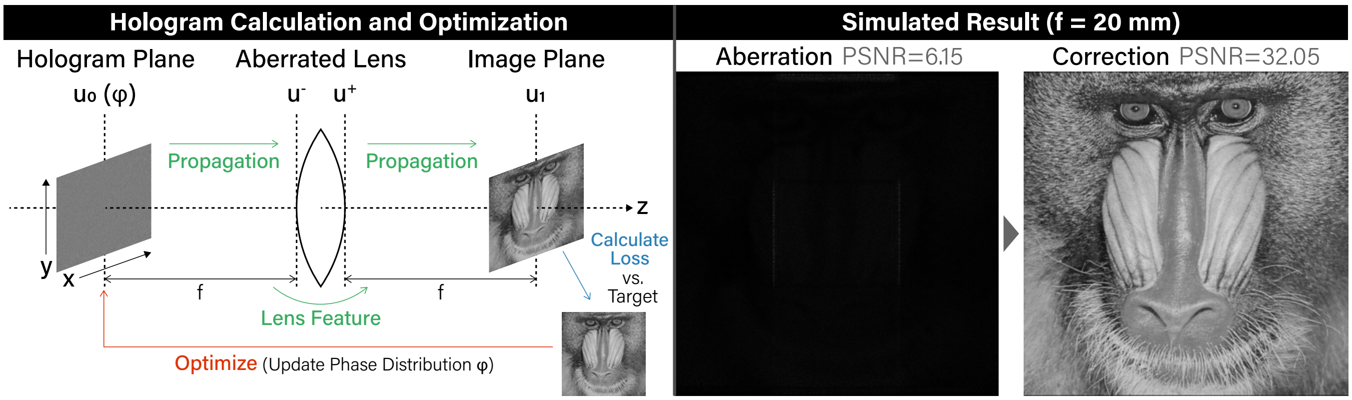

Figure 1: Left: optical system and optimization model. Right: result of our method.

ABSTRACT

We propose a lens aberration correction method for holographic displays via a light wave propagation simulation and optimization algorithm. Aberration correction is an important technology to obtain noise-less hologram images in holographic displays. We optimized phase holograms with an automatic differentiation technique and Adam optimizer, and aberration corrected images were achieved. Given an aberrated lens with a focal length of 20mm, the optimized holographic image has a PSNR value of 32.

CCS CONCEPTS

• **Human-centered computing** → **Displays and imagers**.

KEYWORDS

hologram, optimization, lens aberration

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1 INTRODUCTION

Aberration correction is an essential technology for holographic displays to generate accurate hologram images. In holographic display, image distortion can be caused by wavefront aberration on the lens in the human eye or in the optical hardware, and many methods have been proposed to correct these lens aberrations.

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Takaki *et al.* proposed a Maxwellian-view display that corrects eye aberrations using reversed aberration [Takaki and Fujimoto 2018]. Yamamoto *et al.* proposed an interactive system with a graphical user interface for eye aberration correction by generating hologram images with reversed aberration [Yamamoto et al. 2021].

To correct aberration more accurately, we adopted an optimization technique. Peng *et al.* described that noise-less hologram images can be achieved by optimizing phase distribution on a spatial light modulator (SLM) with an automatic differentiation technique and stochastic gradient descent [Peng et al. 2020]. However, Peng *et al.* did not cover the aberration correction with the optimization.

By applying this hologram optimization method for optical systems with an aberrated lens, we validated the capability of the aberration correction with optimization. We defined the optical system as shown in Figure 1 (left). Light wave propagation calculations and aberrated lens features were included in the loss function, and the phase on the hologram plane of the SLM was optimized to minimize the loss between the intensity distribution of the observed image and that of the target image. We obtained aberration corrected images by this method as shown in Figure 1 (right).

2 METHOD

Computational hologram calculation is based on wave optics [Goodman 2005]. In this study, we calculated light propagation with the angular spectrum method. Propagation from a plane u_a to another plane u_b is represented as follows:

$$u_b = \mathcal{F}^{-1}\left[\mathcal{F}[u_a]\mathcal{F}\left[\frac{z}{i\lambda}\frac{\exp(ikr)}{r^2}\right]\right] \quad (1)$$

where $\mathcal{F}[\cdot]$ is the Fourier transform, $\mathcal{F}^{-1}[\cdot]$ is the inverse Fourier transform, x, y, z are expressed in the coordinate system shown in the Figure 1 (left), i is the imaginary unit, λ is the wavelength, k is the wave number, and $r = \sqrt{x^2 + y^2 + z^2}$. The conversion feature of

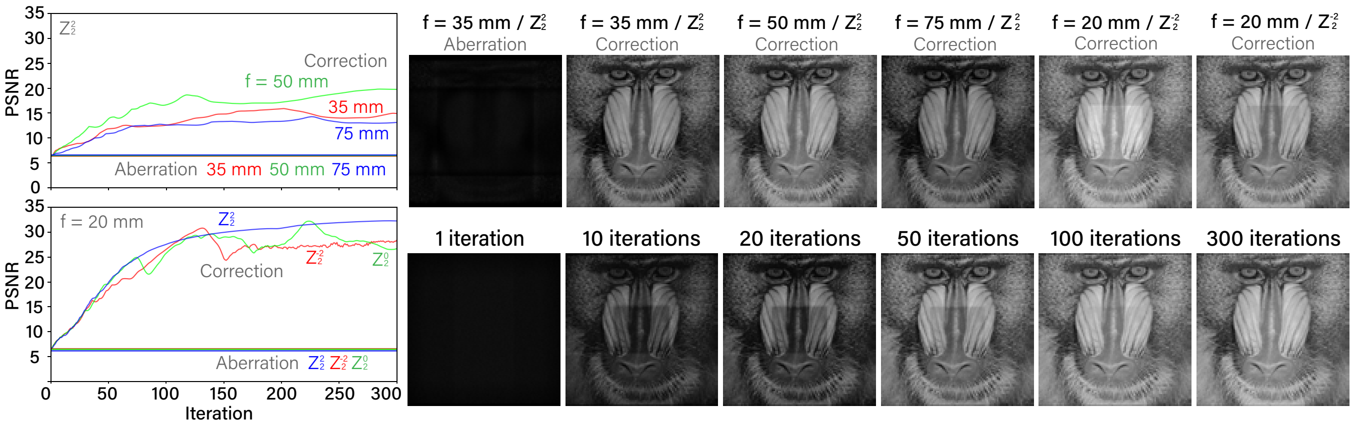

Figure 2: Row 1: relations between iteration and PSNR, and results for lenses with different focal lengths. Row 2: relations between iteration and PSNR, and optimization process.

the lens with aberration can be expressed with the following phase distribution $t(x, y)$ and generalized pupil function $\mathcal{P}(x, y)$:

$$u^{+}(x, y) = u^{-}(x, y) * t(x, y) * \mathcal{P}(x, y) \tag{2}$$

where u^{-} and u^{+} are the complex amplitude distribution at the front and behind the lens, respectively, $t(x, y) = \exp\left(-i\frac{\pi}{\lambda}\left(\frac{x^2+y^2}{f}\right)\right)$ and $\mathcal{P}(x, y) = P(x, y)\exp(ikW(x, y))$. f is the focal length of the lens, $P(x, y)$ is the pupil function and $W(x, y)$ is the wavefront aberration which is generally expressed as a linear combination of the Zernike polynomials [Goodman 2005].

Utilizing Eq.1 and Eq. 2, we formulated our optical system as shown in Figure 1 (left). The image of the object on the image plane is given by the following equation:

$$u_1 = Prop_f\left[Prop_f[u_0] * t(x, y) * \mathcal{P}(x, y)\right] \tag{3}$$

where $Prop_z[\cdot]$ is the propagation computation in Eq. 1, $u_0 = \exp(i\phi(x, y))$ is the hologram plane, and $u_1 \in \mathbb{C}$ is the image plane.

In this system, we calculate the gradient by the automatic differentiation technique and optimized with the Adam optimizer [Kingma and Ba 2014], which is one of the stochastic gradient descent algorithms used to achieve lens aberration correction. We define the loss function as the difference between intensity distribution of the image plane and that of the target image. The loss function in this system is as follows:

$$\mathcal{L}(|u_1|^2, T) = \frac{1}{2}\left(|u_1|^2 - T\right)^2 \tag{4}$$

where T is the intensity distribution of the target image. The optimization process is as follows:

$$\underset{\phi(x,y)}{\text{minimize}} \quad \mathcal{L}(|u_1|^2, T) \tag{5}$$

where $\phi(x, y)$ is the phase distribution of the hologram plane. The image quality is gradually improved while repeating the optimization processes as described above.

3 RESULT

We conducted 300 steps of optimization and propagation simulations on a group of lenses with different focal lengths at constant aberration and a group of lenses with different aberrations with a fixed focal length of 20 mm. Results for these simulations are shown in Figure 2 (row1). In this optical system, aberration makes the resulting images noisy, but applying this optimization technique, we obtained images with reduced noise caused by aberrations.

4 CONCLUSION

This paper proposed an aberration correction method for holographic displays with aberrated lens using automatic differentiation and optimization. By correcting aberration with optimization, Images with less noise were obtained. Application of this method to actual optical system and optimizations for more complicated optical system (e.g. multiple lenses) are planned for future work.

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