Syringe-worked Mermaid:
Computational Fabrication and Stabilization Method for Cartesian Diver

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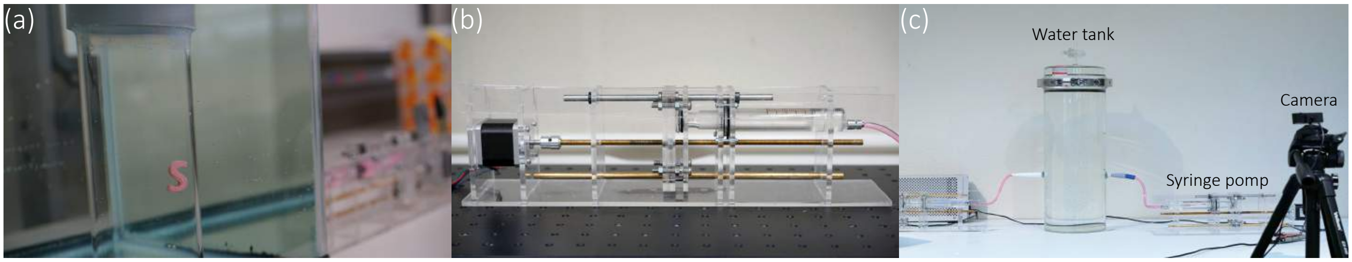

Figure 1: *(a) A Cartesian Diver is moving up and down by moving a syringe pomp. (b) We developed a syringe pomp connected to a stepping motor and controlled by computer. (c) Our system setup. We employed two syringe pomps and connected to a water tank made of grass. And we installed web-camera for PD control.*

Abstract

This paper introduces a new method to add the controllability to underwater objects applied by a structure of a Cartesian Diver which have been computational designed and fabricated. The Cartesian Diver is a well-known demonstration of Pascal's law and Archimedes' principle. The diver is able to be manipulated to achieve up and down motion by a single external force to the primary container based on water pressure changes to the overall volume of the diver or the volume of air in the cavity within the diver. And we discussed the principles and methods to create a digitally designed and fabricated the diver. We then developed several applications and conducted a user study using novice users.

Keywords: Digital fabrication, FEM, underwater interaction, optimization.

Concepts: •Computing methodologies → Shape modeling;

1 Introduction

Underwater entertainment items have been used in many situations: aquariums, fountains, and water tanks displayed in public areas, for example. To study the controllability of underwater objects, we focused on a classic science experiment called the Cartesian Diver. The change in the water pressure within the bottle alters the volume of air in the cavity of the diver. Thus, the diver moves up or down in the bottle by buoyant force. We computationally design Cartesian Diver for connecting digital fabrication and interactive technology.

We aim to connect research on interactive technology counteract gravity to underwater expression. We reviewed literature on digital fabrication using computational design method. Spatial controllability of digital fabrications includes the design and balancing of objects [Prévost et al. 2013]. Some research consider balancing of objects in water tank [Wang and Whiting 2016; Musialski et al. 2015; Wu et al. 2016].

2 Implementation and Method

Our method consists two phases; fabrication and stabilization(Figure 2). In fabrication phase, we design a Cartesian Diver structure to hollow cavity inside an objects. To establish such structure, it is necessary that upward force applied to the object is larger than downward one at the moment that floated in the water. In addition it, we need to avoid causing rotational moment to the objects because it is required that we make a hole the models bottom and keep it downward.

At first, we use the equation motion to obtain amount of cavity which is lowest limit for floating:

$$m\alpha = mg - \rho V_{out} g - \rho V_{in} g - \frac{1}{2} C_d \rho v^2 \tag{1}$$

where m is a diver's mass, α is diver's acceleration, g is a gravity acceleration, V_{out} is volume of the object itself, V_{in} is volume of the cavity, C_d is resistance coefficient, ρ is water density, v is a characteristic velocity. When the diver is inserted in water(time t = 0), both α and v are assumed to equal zero. Therefore, equation(1) becomes $0 = mg - \rho V_{out} g - \rho V_{in} g$. From this equation, it is obvious that upward force and downward force are applied to the diver at the moment that floated in the water; two buoyancy and a gravity. To fabricate a Cartesian Diver, it is set at top of the bottle at first (t = 0), so it is necessarily that upward force is larger than downward one. Then, $\rho V_{out} g + \rho V_{in} g > mg$. From this equation, we obtain amount of cavity that should be hollow inside the object. In a next step, we optimize a position of cavity. We utilize geometrically non-linear finite element analysis in voxelization method to optimize it. The object in water tank is inclined against three axis; x, y, z (like roll, pitch and yaw). In this case, we consider two axis; axis x and axis y because axis z has no effect direction of bottom and is too difficult to control by buoyancy and pressure. So we analysis these two kinds of axis, axis x and axis y, and optimize

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SA '16, December 05-08, 2016, , Macao
ISBN: 978-1-4503-4540-8/16/12
DOI: http://dx.doi.org/10.1145/3005274.3005316

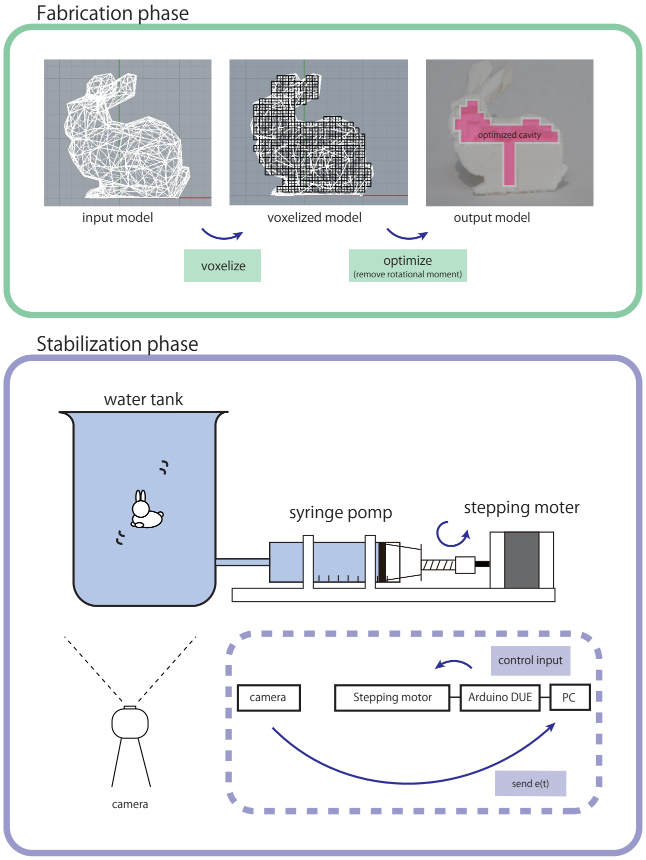

Figure 2: *Fabrication phase pipeline(upper) and stabilization phase pipeline(bottom).*

position of holes. We formulate as follows about angular moment:

$$M_y = \int (x - x_0) \left(\int (mg - \rho V g) d_y \right) d_x \tag{2}$$

$$M_x = \int (y - y_0) \left(\int (mg - \rho V g) d_x \right) d_y \tag{3}$$

where M_x and M_y is angular moment around axis x and y, which moment center position is $(x_0, y_0, 0)$. x, y and z is position of each voxels. m is a voxel's mass. g is a gravity. In stabilization phase, we provide a Cartesian Diver manipulation system for controlling the buoyant force inside the diver and employ PD control to stabilize the object in the middle of water tank. In order to use PD control, we need 2 values difference $e(t)$ between a target spot and a present position. From $e(t)$:

$$N = -K_p e(t) - K_d \frac{de(t)}{dt} e(t) \tag{4}$$

Where N is control input; rotational speed of the stepping motor, K_p is a factor of proportionality and K_d is differential constant. Syringe pomp moves a mm per 1 motor rotation:

$$\Delta V_N = S * Na \tag{5}$$

Where ΔV_N is the volume of change per N motor rotations. Since the diver's buoyancy changes $\rho NaSg$ per N motor rotations repeatedly, we control the diver. We also provide a Cartesian Diver manipulation system for PD control. Shown in Figure 1, the bottle filled with water is connected to a syringe pomp and tube which are also filled with water. To track the position of the diver, we installed a camera in front of the bottle. Using a stepping motor connected to an Arduino system, the syringe pomp is moved forward or backward and controls the amount of water. This action results in a change of the buoyant force inside the diver.

Figure 3: *An application example. Cartesian Divers shaped letters.*

3 Application

In this paper, we reported on a computational design for Cartesian diver to extend traditional its methods. The method has wide-ranging applications, such as multiple divers and the ability to use projections inside a tank of water. We implemented these applications using our system, conducted experimental evaluations, and demonstrated the capabilities of the system.

Figure 3 shows an application example. Conventional Cartesian divers appear to be similar as there are not many varieties of the form. Using our method, it becomes possible to fabricate Cartesian divers having different forms, yet control the buoyant force as with a conventional diver. We fabricated several types of Cartesian divers such as a bunny and the letters S G A H for experimental use in the test bottle. For the situation where manipulation of multiple divers is required, we designed the order the divers are immersed. The volume of air was altered through the location of the cavity. The design of the cavity is detailed in the 3D model and during fabrication is output by the 3D printer. By utilizing this approach, we can control the immersion timing of multiple divers.

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